Handling contradictions in learner modelling in the "Translating Algebraic Problems System" (TAPS)[1]

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Abstract: This paper describes how the Translating Algebraic Problems System (TAPS) deals with learners' misconceptions related to the *reversed equation*. Such learners' misconceptions are caused by inconsistency and contradiction of their own beliefs, which are also entrenched in their reasoning. Revision of conflicting beliefs which arise when TAPS executes its meta-level observation of the learner's overall performance is also discussed.

Translating and formulating an equation from a simple relational algebra word problem (RAWP) is not easy for many people. Among the typical errors that are often made by many of us in translating such problems is formulating a *reversed equation*, as illustrated in Figure 1 and discussed by (Lochhead & Mestre, 1988) and (Rosnick & Clement, 1980). This error is also referred to as the *reversal error*. It has been observed that some of the misconceptions which lead to the reversed equation are caused by inconsistency and contradiction of their own beliefs in reasoning. The Translating Algebraic Problems System (TAPS) is a prototype of a computer-based tutoring system which allows learners to practice the translation process and hence improve their skills in translating RAWPs to algebraic equations. In doing so, contradictory belief due to the nature of the reversal error in RAWP and conflicting information while modelling a learner can arise during an online session with TAPS.

This paper will describe how TAPS deals with learners' misconceptions related to the reversed equation. Such learners' misconceptions are caused by inconsistency and contradiction of their own beliefs, which are also entrenched in their reasoning. Revision of conflicting beliefs which arise when TAPS performs its meta-level observation of the learner's overall performance is also discussed.

Belief revision and learner modelling

Belief revision (also called *truth maintenance* or *reason maintenance*) is an area in Artificial Intelligence where sets of beliefs are revised to maintain consistency of information in a system. Such an action is taken by a belief revision system when new information contradicts the existing information. The system resolves the contradiction between two beliefs to maintain the consistency of information in a knowledge base. Knowledge could be described in terms of beliefs and be applied in learner modelling (Self, 1992).

Learner modelling in a tutoring system provides information about what the learner understands or does not understand. This enables the system to to provide individualised teaching according to the learner's capability and needs. Among the important issues in learner modelling as discussed by (Mizoguchi, 1993) is the capability of handling inconsistency of information. In reality, learners' beliefs may or may not be consistent with what is expected by the tutor. A conflict in terms of contradiction may occur here. At the same time, while modelling a learner, the tutor's belief about the learner at one time may contradict its (the tutor's) former belief. Such a conflict of information or beliefs happens when learners translate a RAWP and formulate a reversed equation from it. TAPS uses a belief revision system in its learner modelling to handle the occurrence of conflict information.

During a visit to an old university, the tour leader said that, *'there are 6 times as many students as professors in this university.'* If S is the number of students and P the number of professors, the algebraic equation one should formulate from the above statement is

$6S = P$ (typical incorrect answer) $S = 6P$ (correct answer)

Figure 1: Example of a relational algebra word problem where many people translate and formulate the reversal of the correct equation

Relational algebra word problems and the resilient misconceptions

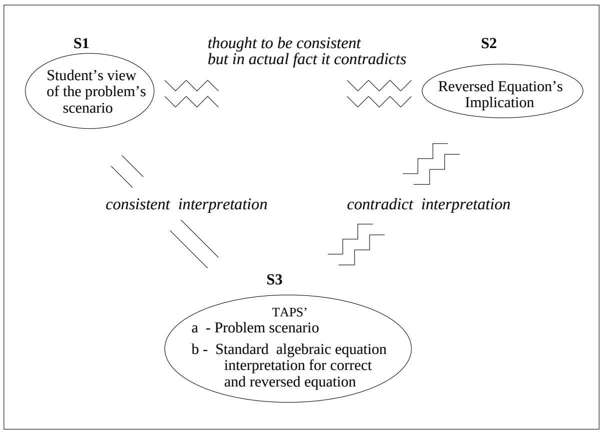
TAPS handles relational algebra word problems in the form of problem statements such as that in Figure 1. Translation of such RAWP involves translating the words of the problem into an internal representation in memory, and then formulating an equation out of it, as defined by (Paige & Simon, 1966) and (Mayer, 1982). If a solution is needed then one has to apply the legal rules of algebra and arithmetic to this internal representation in order to deduce the answer. The major difficulty lies in the translation phase although most instruction (in many education system) focuses on the solution phase (Simon, 1980). (Clement, Lochhead, & Soloway, 1980) found that the most common mistake made by students is due to their confusion about the semantics of the algebraic equation. This then leads the students to adopt the incorrect strategy when translating the sentence and thus formulate the reversed equation. The learner's misconceptions related to the reversal error are usually resilient and difficult to dislodge from his reasoning process (Lochhead & Mestre, 1988). These resilient misconceptions are beliefs entrenched in the learner's mind. Such a belief is difficult to drop and be replaced by a new belief. It resurfaces from time to time in the believer's reasoning process either consciously or unconsciously. This problem have been found to be widespread across age, academic background, and nationality (Aziz, Pain, & Brna, 1995). (Brewer & Chinn, 1991)'s study illustrates how people deal with information that violates their entrenched beliefs: *reject the information; reinterpret the information;* or *make partial changes in their former beliefs through accommodation and/or compartmentalization.* Similarly in the case of translating RAWP, several behaviours which illustrate a learner's entrenched misconceptions have been observed by (Rosnick & Clement, 1980). Among them are *the learner reverts back to the reversed equation; the learner accepts the correct equation but switches the meaning of the original problem*; and *the learner identifies the correct equation as "not making sense"*. TAPS combines both of (Rosnick & Clement, 1980)'s justifications and (Brewer & Chinn, 1991)'s findings to capture the learner's entrenched misconceptions during the teaching session.

The more tenacious beliefs are usually more central to the reasoner's whole system of beliefs compared to the less tenacious ones. These entrenched beliefs are also more useful for the reasoner's general style of reasoning and argumentation, as noted by (Berendt & Smaill, 1992). Hence, we find that it is important for TAPS to apply a belief revision system in assisting the learner to elicit his tenacious incorrect beliefs (misconceptions) and replace such beliefs with the correct ones. This has been further discussed in (Aziz et al., 1995).

Translating Algebraic Problems System (TAPS)

TAPS is intended to tutor learners with difficulties in the *translation of algebra word problems.* The particular aspect we are concern with is the learner's deeply-seated or entrenched misconceptions. There are three stages in the learner's interaction with TAPS: pre-test, the teaching session and post-test. Based on the learner's mastery level determined in his pre-test, TAPS generates a dialogue session between TAPS and the learner (multiple choice question-answer session). The questions and instructions used are carefully designed by considering possible learner's reactions to entrenched beliefs and ways of provoking and entrapping the learner's beliefs. For this, integration of (Rosnick & Clement, 1980)'s and (Brewer & Chinn, 1991)'s works have been applied as mentioned in the previous section.

TAPS applies the *cascaded problem sets teaching architecture* (Schank, 1990). The idea of the cascaded problem sets architecture, is to build a problem space where each problem is related to the next problem by an extra layer of complexity. (Schank, 1990) suggested that the libraries of cascades built could be done by breaking up a problem into its constituent parts. Each constituent would itself be a problem, and it too would have constituent parts. This cascade of problems would continue until a basic level of related knowledge is reached. In TAPS, a cascade is a set of related questions for one relational algebra word problem. Each cascade is decomposed into constituent parts which are the abstract properties of the



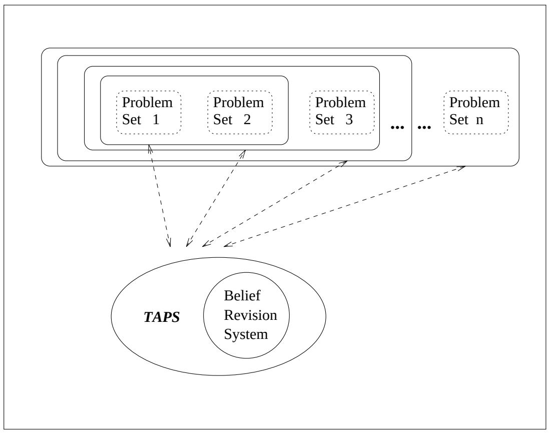


Figure 2: Level 1 conflict - occurs when a student formulates a Reversed Equation

Figure 3: Level 2 conflict - TAPS' observation on the student's overall performance

questions. Examples of the abstract properties are *correct quantitative-qualitative understanding, reject the word order approach, reverse the meaning of the RAWP* and *apply the correct translation strategy.* The questions are assessed from a pre-stored question library.

TAPS and contradictions in translating RAWP

We define a conflict as believing both a concept C and $\neg C$ at the same time or in other words a contradiction of beliefs. The terms conflict and contradiction refers to the same meaning in our discussion. There are two levels of conflict in TAPS — *level 1 conflict* and *level 2 conflict.* In this paper, we will refer to *level 1 conflict* and *level 2 conflict* as the *triangle conflict* and *meta-level observation conflict* respectively. TAPS seeks to derive contradictions which take place: *i)* within the learner's own beliefs; *ii)* between the learner's and TAPS's beliefs about the domain; and *iii)* among TAPS's own beliefs about the learner. *i)* and *ii)* usually occur in the *triangle conflict* during the learner's diagnostic session, whereas *iii)* arises in the meta-level conflict during TAPS' learner remediation process.

In the *triangle conflict*, contradictions occur within the learner's reasoning process, that is between his qualitative-quantitative understanding and his conceptual understanding of the reversed equation he formulated. For example, in the sentence *"There are 5 times as many palm trees as coconut trees that dad had grown on the piece of land"*, a learner believes qualitatively that there are more palm trees than coconut trees. His formulated equation $5P = C$, (P as the number of palm trees and C as the number of coconut trees) which is the reversal of the correct equation ($5C = P$) implies the opposite situation i.e there are more coconut trees than palm trees. This contradiction is between the learner's qualitative-quantitative understanding and that of the standard algebra implication of the reversed equation. This is described in Figure 2 as *S1 contradicts S2* which will be indicated as $S1 \perp S2$. A conflict also occur when the learner's understanding of the reversed equation's and the correct equation's implication contradicts TAPS's own implications of both equations according to the standard algebra interpretation. Thus, *(S1 and S2) contradicts S3b* which will be indicated as $(S1\&S2) \perp S3b$.

In TAPS' *meta-level observation conflict*, the learner's beliefs (the misconceptions) can be deeply entrenched in his reasoning. The learner may fail to be consistent over time due to the resurfacing of his tenacious misconceptions. This causes TAPS's beliefs that "the learner is capable of translating RAWP" to contradict with "the learner is incapable of translating RAWP". Here, TAPS's belief about the learner's capability contradicts with TAPS' another belief on the learner's performance, over time. TAPS' observation compares the learner's performance in the current set of problems he is doing with his performance in the previous problem sets he had gone through as indicated by the solid boxes in Figure 3. This conflict situation may occur when TAPS evaluates the learner's overall performance.

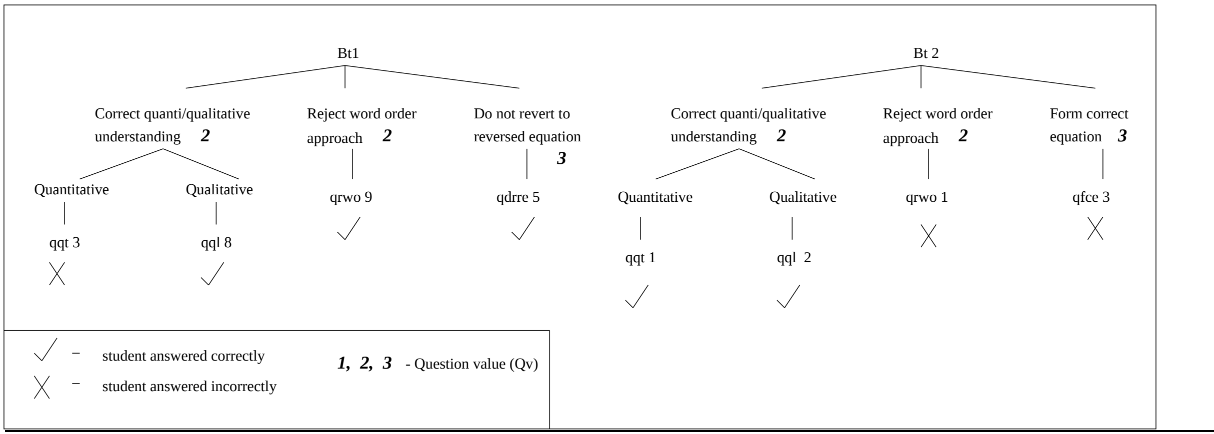


Figure 4: Evaluating the question values for conflict between *"the learner is capable of translating RAWP"* and *"the learner is not capable of translating RAWP"*

Integrating belief revision and learner modelling in TAPS

A modified version of Berendt's *Belief Revision* system (BR) (Berendt, 1992) is employed in TAPS' learner modelling. Berendt's BR system is a *Truth Maintenance System* (TMS) with a combination of the foundational and the coherence approaches. A logic-based formalism is used to represent the beliefs. The beliefs are modelled as statements in (a restricted) first-order logic. The meta-theory used in modelling its deductive process is discussed further in (Berendt & Smaill, 1992). Contradiction is resolved by the preference ordering of beliefs which is based on Gärdenfors' notion of *epistemic entrenchment* (Gärdenfors, 1988). Here, the two beliefs involved in a contradiction are revised based on their *strength of belief*, the beliefs' entrenchment values. The entrenchment value represent the validity of the belief. The more entrenched (higher value) belief and its justifications will retain while the less one with its justifications will be dropped from the belief set.

The BR system is used in TAPS to detect and resolve the two levels of contradiction that occur in TAPS (as described in Figures 2 and 3). TAPS resolves a contradiction through BR's application of the epistemic entrenchment notion and invoking TAPS' decision rule procedure to determine which subset of beliefs is to survive when a conflict arises. TAPS uses justifications of the contradicting beliefs for explanation during the remediation session. Using a learner's own beliefs in the explanation provides information which he is familiar with. As highlighted by (Kass & Finin, 1988), familiarity is one of the characteristics of understandable explanations.

Three values of entrenchment are used in TAPS: i) *Tev* - TAPS's entrenchment value. Implication of the learner's beliefs by TAPS also uses *Tev* ; ii) *Sev* - Student's entrenchment value; iii) *Tov* - TAPS's observation's (meta level) entrenchment value.

In the triangle conflict, the competition of beliefs is between *Tev* and *Sev*. *Tev* is always greater than *Sev* since ideally beliefs (knowledge) of TAPS about the domain are always assumed to be correct. Hence, the initial belief (that leads to a contradiction) with *Tev* will be regarded as more valid and will survive in the competition, while the one with *Sev* will be discarded. This revision process occurs in the BR system. BR will then notify TAPS that the disqualified belief and its justifications are the ones that the learner has to withdraw from his reasoning process.

In the meta-level observation conflict, however, the competition of the beliefs now is between two *Tov*s where both have the same entrenchment value. Berendt's BR system applies Gärdenfors' *conservative* way of choosing the belief that is to be kept when resolving a conflict. In this approach, the earlier belief is considered to be more reliable compared with the newer ones. Unfortunately, this is not the case during a teaching session. An earlier observation should not necessarily be assumed to be more valid than a recent one. For this, we evaluate the abstract properties of the questions that the learner has been asked and relate these entities to *Tov*. The ones with stronger justifications will not have their entrenchment value decremented and thus survive in the belief set. Each question being asked in the teaching session holds a value of difficulty. We call this the *question value* (Qv). Three categories of

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T: 'There are 7 times as many children as adults at the playground.'
   The number of adults are ____ the number of children at the playground
   a. more than    b. less than    c. the same as
L: b
T: To make the number of children (C) more than the number of adults (A), we should multiply 7 to C,
   so that C will be more than A. Yes/No ?
L: y
                          >< >< ><  CONFLICT DETECTED  >< >< ><
Aha ... there's a conflict here.
You seem to believe 'there are more children than adults' AND ALSO
'there are more adults than children' at the same time.
The belief - 'there are more adults than children' arises from:
1. Your answer - 'multiply 7 to C, so that C will be more' which leads to 'the coefficient 7 is
being associated with C to make C as the larger group' which leads to 'a reversal of the correct
equation' which then leads to 'there are more adults than children'.
The belief - 'there are more children than adults' arises from:
1. Your answer - 'the number of adults are less than the number of children' which leads to
   'there are more children than adults'  2. According to TAPS - 'there are more children than adults'.
Now, do you want to:
a. get further explanation  or   b. fix the answer
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Figure 5: A triangle conflict had been diagnosed. Part of the explanation is shown here as the remediation session to the learner.

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                          >< >< ><  CONFLICT DETECTED  >< >< ><
The belief - 'the learner is not capable to translate RAWP' arises from:
1. According to TAPS - 'the learner has contradict beliefs and he also scores below average in problem
set 1' which leads to 'the learner does not perform well and not capable to translate RAWP'
The belief - 'the learner is capable to translate RAWP' arises from:
1. According to TAPS - 'the learner does not have contradict beliefs and he scores above average in
problem set 6' which leads to 'the learner is capable to translate RAWP'
Looking back at the meta-level observation, the belief 'the learner is able to translate RAWP and its
underlying beliefs are more justified. However the opposite belief 'the learner is not capable to
translate RAWP' and its causes which due to the learner's answers in :
Problem Set 1 - Question 5 and Problem Set 1 - Question 4 seems to be a bit of a problem to the learner ...
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Figure 6: A meta-level observation conflict and part of its explanation. This information is recorded in a separate file for the teacher's reference.

questions are formed: *i)* simple, with the Qv of 1, *ii)* moderate, with the Qv of 2, and *iii)* tricky, with the Qv of 3. When competition between two *Tov*s occurs, TAPS will refer to the related justification from BR's backward chaining of reasoning which ends up with the beliefs identifying the learner's answer to the related question. TAPS will access the questions involved and examine whether the learner answers each of these questions correctly or incorrectly. Only those answered correctly will be taken into account i.e their Qv will be added up to produce the total value as *Tqv1*. The same evaluation is done for the other competing belief which will result in *Tqv2*. These two values (*Tqv1* and *Tqv2*) are compared. The one weighted heavier will have its related *Tov* incremented.

An example as shown in Figure 4 is described as follows. A contradiction of TAPS believing that *(the learner is capable of translating RAWP)*, referred to as B_{t1} and *(the learner is incapable of translating RAWP)*, referred to as B_{t2} occurs. The questions involved in producing B_{t1} are qqt3 (question on quantitative understanding number 3), qql8 (question on qualitative understanding number 8), qrwo9 (question on rejecting word order approach number 9), and qdrre5 (question on do not revert to reversed equation number 5). The learner answered correctly for qql8, qrwo9 and qdrre5. Only those correctly answered will have their Qv counted. In this case, the *correct quantitative-qualitative understanding* property is not counted since only qql8 is correctly answered but not qqt3. Hence, B_{t1} has its Tqv_1 with the value of 5. Tqv_2 for B_{t2} is only 2. Since Tqv_1 is greater than Tqv_2, B_{t1} will be the one favoured by the BR system to survive in the contradiction. However, TAPS having gone through the evaluation of Qv will take note that the learner is *"able to translate RAWP"* but the learner still needs more practice on questions that he had made mistakes on. His following remediation will focus on the questions which he had answered incorrectly. Figure 5 and 6 are excerpts of TAPS online session where a triangle conflict and a meta-level conflict had occurred and their related justifications are used as contents of explanation.

Table 1: Some issues considered in THEMIS and TAPS

Issues in THEMIS	Related matters in TAPS
1. 4 categories of contradiction: a. a learner changes his mind, causing inconsistency in his answers [A1] b. slips which caused oracle contradiction [A2] c. learner knowledge contradiction in his reasoning process [A3] d. assumption contradiction in modelling [B]	1. consideration taken as: a. resurfacing of misconceptions by learner. b. handled by repetition of isomorphic questions. c. captured in the triangle conflict $S1 \perp S2$. d. captured in: i) triangle conflict - $(S1\&S2) \perp S3b$ ii) meta-level observation conflict.
2. single world and multi-world contradictions	2. contradiction occurs as: i) single world in ($S1 \perp S2$), (between learner's beliefs) and meta-level observation conflict (between TAPS' own beliefs) ii) multi-world in $(S1\&S2) \perp S3b$, (between learner's and TAPS beliefs)
3. formulate learner's knowledge contradiction based on the concept discrimination structure and multi-world logic	3. formulate both learner's & TAPS's knowledge contradiction based on BR's meta-theory and TAPS' overlay learner modelling
4. calculates the certainty of each clause in its method level learner model by referring various information	4. calculates the question values that are related to the abstract properties of the questions which the learner has answered

Related work

The issue of maintaining consistent information in learner modelling has been raised among others by (Woolf & Murray, 1993), and (Self, 1992). A small but significant amount of research such as (Huang, 1993), (Paiva, Self, & Hartley, 1994) and (Kono, Ikeda, & Mizoguchi, 1994) deals with maintaining consistency of information in learner modelling. The recent discussion of (Kono et al., 1994) on THEMIS seems to be the most relevant to our work. THEMIS is a nonmonotonic and inductive model inference system which includes the Hypothetical Student Modelling Inference System (HSMIS). This incorporates deKleer's Assumption-based Truth Maintenance System (ATMS) to maintain consistency of the learner modelling process. Both TAPS and THEMIS view learner modelling as inductive learning from sets of examples. TAPS and THEMIS consider maintaining consistent information while modelling a learner as an important component in our systems. The issues which both systems consider are summarised in Table 1 and discussed as follows.

THEMIS classifies its student contradictions into 4 types. Each of these types are also considered in TAPS. THEMIS' A1 type of contradiction (when a student changes his mind and causes inconsistency in his answers) is what TAPS considers as the resurfacing of misconceptions in RAWP. TAPS can diagnose this when a meta-level observation conflict occurs. In THEMIS, a student's answer is represented in an oracle. An oracle consists of a fact and its truth value. (Kono et al., 1994) refer to the student's slips which caused oracle contradiction as A2. TAPS tries to detect the student's slip errors by having isomorphic questions repeated and allowing him to view and change his answers after each set of questions.

A contradiction in a student's reasoning process (THEMIS' A3) is captured in TAPS' triangle conflict i.e $S1 \perp S2$. However, (Kono et al., 1994) argue that this type of contradiction should not be resolved, but such contradictory knowledge should be represented as it is, for effective tutoring. Besides representing the learner's contradictory knowledge as it is, we find that it is useful to resolve the conflict in TAPS. Resolving by revising the conflict beliefs provides us the justifications involved and the chain of reasoning that causes the contradiction. This information is then used as explanation to the learner (for remediation) in the teaching session. Resolving the contradiction here also allows detection of a conflict between the same beliefs, if it arises again. This is relevant in particular for resurfacing misconceptions. In terms of THEMIS' assumption (type B) contradiction, TAPS identifies and takes care of this in its triangle conflict, $(S1\&S2) \perp S3b$ and also in the meta-level observation conflict. THEMIS emphasizes its differentiation of the single world and multi world contradictions. This is due to its different approach of handling each dimension of contradiction. Both single and multi world contradictions occur in TAPS too. The single world happens in the triangle conflict of ($S1 \perp S2$), i.e between the learner's own beliefs, and in the meta-level observation conflict i.e between TAPS' own beliefs. As for the multi world conflict, this arises in the triangle level conflict of $(S1\&S2) \perp S3b$ i.e between the learner's and TAPS' beliefs. TAPS however does not differentiate the single and multi world contradiction since both dimensions of conflicts can be solved in similar ways by BR. Beliefs that belong to either the learner or the tutor (TAPS) can be distinguished by BR's backchaining of the self-evident assumptions. These assumptions indicate whether it is the learner's or TAPS' beliefs. Examples of BR's self-evident assumptions are *answers(student, X)*, and *says(TAPS, Y)*, which indicates the believer of the belief proposition. THEMIS formulates a student's knowledge contradiction based on the concept discrimination structure and multi-world logic (Kono et al., 1994). TAPS formulates the learner's knowledge contradiction by combining the overlay learner modelling method and BR's formal representation and deductive process.A similar issue for evaluating and calculating some values based on the learner's previous information arises during the process of resolving a contradiction. THEMIS' calculation refers to various sources of information such as the number of top-level traces that justify the clauses, whether the oracles of the clause are correct answers or not, and how old the oracles are (Kono et al., 1994). However, TAPS calculation only refers to the abstract values of the questions the learner had answered and whether the questions were answered correctly or not. This is sufficient and feasible for TAPS since it deals with a specific restricted domain. The features of the cascaded problem sets architecture which have been discussed, helps TAPS to minimize the sources of information TAPS needs to reference.

The discussion in this section has highlighted some similarities and different issues that arise when developing a learner model which utilises a reasoning maintenance system. THEMIS, being part of FITS (Framework for ITS) (Ikeda & Mizoguchi, 1994) is a complex and large system which is developed as a domain independent system. (Kono et al., 1994) have considered many fine grain aspects in diagnosing and resolving conflicting information in its student modelling. TAPS on the other hand, concentrates on a small specific domain, relational algebra word problems. Its approach in the various issues above is believed to be adequate for TAPS' research objective i.e to illustrate the potential of belief revision in a tutoring system for a domain where contradictions arise when learners commit the reversal error. A comparison in performance between THEMIS and TAPS is not feasible currently. It remains to be seen which is more effective.

Summary and Conclusion

This paper has described the misconceptions involved when learners produce reversed equation in the domain of relational algebra word problems. We indicated two levels of conflicts that could occur in TAPS' learner modelling. TAPS tackles these contradictions by employing the BR system. The mechanism for diagnosing and resolving the conflicts which occur has been illustrated. Comparison and contrast issues were considered in relation to THEMIS (Kono et al., 1994), highlighting the relevant subjects worth considering when one employs belief revision in learner modelling.

As mentioned earlier in this paper, many people find algebra problems with relational propositions difficult to translate. This applies not only to translating from i) words to algebraic equation, but also to ii) writing a sentence from a given two-variable linear equation; iii) forming an equation representing the relationship between 2 variables from a tabular format; and iv) producing an equation describing the relationship between 2 variables from a pictorial format. The most common error is to produce the reverse of the correct equation for cases i), iii) and iv) and the reverse meaning of the sentence in case ii) as discussed in (Aziz, 1993). TAPS currently deals with case i) only. It will be interesting to extend TAPS to handle cases ii), iii) and iv).

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