Modelling & mending students' misconceptions in translating algebra word problems using a belief revision system in TAPS [1]

Normaziah A. Aziz, Helen Pain
Department of Artificial Intelligence
University of Edinburgh, 80, South Bridge, Edinburgh, EH1 1HN, UK
E-Mail: maziah@aisb.ed.ac.uk

Paul Brna
Department of Computing
University of Lancaster, Lancaster LA1 4YR, UK

Abstract: The goal of this research is to illustrate the potential of belief revision in a tutoring system for a domain where contradictions arise when students commit errors. We show how a belief revision system which employs the notion of epistemic entrenchment in resolving a conflict can be useful in modelling students and for remediation. Our particular focus is on: i) the domain of translating simple relational algebra word problems to algebraic equations; ii) misconceptions related to the reversal error; iii) student being persistent in holding the misconceptions which are referred to as the student's entrenched beliefs. We illustrate such investigations in TAPS *(Translating Algebraic Problems System)*. TAPS, a prototype computer-based tutoring system has a belief revision system embedded in it which assists it in modelling and mending a student's misconceptions. It also helps TAPS in making overall observations of the student's performance.

Several empirical studies have observed that many students face difficulties in translating algebra word problems with sentences that state relationships between two variables, e.g. (Mayer, 1982) and (Paige & Simon, 1966). Among the typical errors that are often made in translating such problems is formulating a *reversed equation*, as illustrated in Figure 1 and discussed by (Lochhead & Mestre, 1988) and (Rosnick, 1981). This error is referred to as the *reversal error*. It has been observed that some of the students' misconceptions which lead to the reversal error are caused by inconsistency and contradiction of their beliefs in reasoning. This is further discussed in the third section of this paper.

Belief revision deals with contradictions of information or beliefs in the light of new evidence. A belief revision system resolves the contradiction between beliefs to maintain the consistency of beliefs in a knowledge base. As knowledge can be described in terms of beliefs, belief systems have been applied in student modelling, as discussed in (Self, 1992). Student modelling deals with dynamic environments particularly in relation to the student's behaviour. Behaviour in this context is defined by (Dillenbourg & Self, 1992) as "a sequence of actions performed by the student as a potential solution for some problem". The term *student's belief* in this present paper will refer to the definition of behaviour here. In reality, students' beliefs may or may not be consistent with what is expected by the tutor. A contradiction can occur here. At the same time, while modelling a student, the tutor's belief about the student at one time may contradict its (the tutor) former belief.

The student's misconceptions related to the reversal error are usually deeply seated and difficult to dislodge (Lochhead & Mestre, 1988). These misconceptions have been found to be widespread across academic background, age, and nationality (Clement, 1982) and (Mestre & Lochhead, 1983). A further recent survey of adults translating algebraic problems (words, tables and picture) into algebraic equation carried out by the first author supports this. The present work weaves together a belief revision system

1. *There are six times as many students as professors in the university.*
   Typical incorrect answer: $6S = P$ Correct answer : $S = 6P$
2. *At Mindy's restaurant, for every four people who order cheesecake, there are five people who order strudel.*
   Typical incorrect answer: $4C = 5S$ Correct answer : $5C = 4S$

Figure 1: Examples of relational algebra word problems where students produce reversed equation when asked to translate from sentence to equation

and student modelling, to diagnose and mend the misconceptions that lead to the reversal error. The purpose is to explore the integration and potential of belief systems in *modelling for remediation* of entrenched misconceptions in the domain of translating algebraic problems.

Relational algebra word problems

An algebraic word problem consists of one or more sentences having some known or unknown values with underlying algebraic equations. The algebraic word problems we are discussing here are the ones with relational propositions such as that in the *student-professor* word problem of Figure 1.

Two cognitive processes are said to be involved in solving story (word) problems (Paige & Simon, 1966) and (Mayer, 1982): *i)* Translation - understanding the problem, as manifested in translating the words of the problem into an internal representation in memory; and *ii)* Solution - applying the legal rules of algebra and arithmetic to this internal representation in order to deduce the answer. (Simon, 1980) stated that the major difficulty lies in the translation phase although most instruction focuses on the solution phase. As for translating the *student-professor* type of relational word problems, (Clement, Lochhead, & Soloway, 1980) found that the difficulty faced by the students is due to their confusion about the semantics of the algebraic equation.

A number of common concepts and misconceptions that many students encounter while translating the *student-professor* type of relational word problem to an equation have been identified (Rosnick & Clement, 1981); and (Clement, 1982). Such misconceptions then lead the students to adopting the incorrect strategy while translating the sentence. Some of the common incorrect strategies applied by the students are: the *word order matching* or *direct mapping* approach and the *static comparison* method. On the other hand, the common correct strategies which lead the student to formulate the correct equation are: the *operative* approach and the *ratio* approach. The description of these strategies is given in Table 1.

Table 1: Common approaches that students apply in translating and formulating relational algebra word problems

Strategies	Description
1. Word order matching	The student maps directly from the order of the keywords in the problem statement. This is done syntacticly without understanding the meaning of the equation formulated.
2. Static comparison	The student places multiplier next to the letter associated with the larger group. Lack of understanding on the functions of the variable, the coefficient and the equal sign is the underlying reason for this.
3. Operative	The student views the equation as representing an operation on a variable quantity to produce a number equal to another unspecified quantity.
4. Ratio	The student forms the ratio on both groups involved in the problem statement. Then a cross-multiply operation between the variables and the ratio values is performed.

Features of student's misconceptions in TAPS

TAPS seeks to derive contradictions that take place *i)* within the student's own beliefs; *ii)* between the student's and TAPS' beliefs about the domain; and *iii)* among TAPS' own beliefs about the student. The contradictions can be illustrated in 2 levels:

First, the student is unaware of the contradiction between the implication of his beliefs, and that of the standard algebraic interpretation. This will be referred as a **Level 1 conflict** and is illustrated in Figure

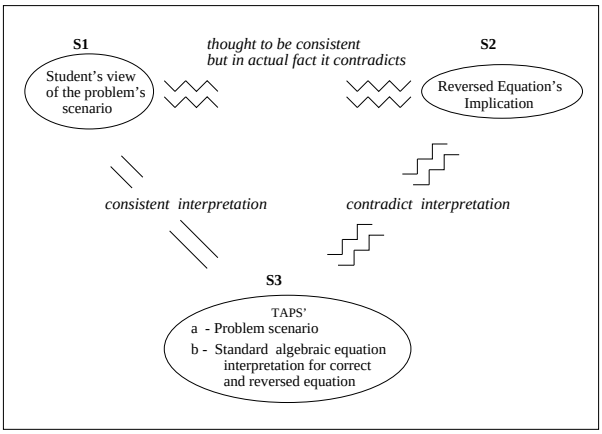

Figure 2: Level 1 conflict - occurs when a student formulates a Reversed Equation

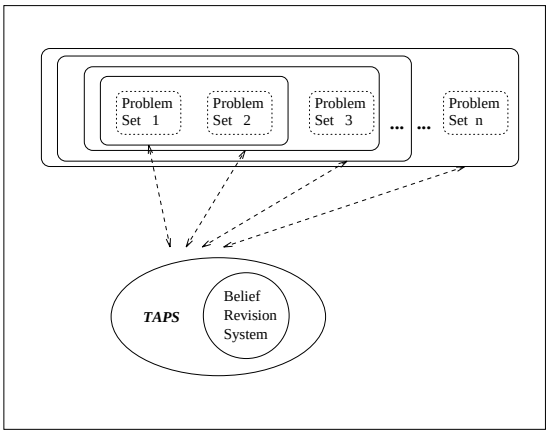

Figure 3: Level 2 conflict - TAPS' observation on the student's overall performance

2. In Level 1 conflict, the student's view of the problem scenario (his qualitative and quantitative understanding) is consistent with TAPS' view of it. That is *S1 is consistent with S3-a*. However, contradictions do occur within the student's mind, that is between his qualitative-quantitative understanding and his conceptual understanding of the reversed equation he formulated. His thought that *object-a* is more than *object-b* contradicts with what his reversed equation implies. The student is usually unaware that such a contradiction exists.[2] For example, in the sentence *"There are 5 times as many children as teachers in the kindergarten"*, students believe qualitatively that there are more children than teachers. His formulated equation which is a reversal equation implies the opposite situation i.e there are more teachers than children. Hence, *S1 contradicts S2*. Contradictions also occur when the student's understanding of the reversed equation's and correct equation's implication contradicts with TAPS' own implications (of both equations) according to the standard algebra interpretation. Thus, *(S1 and S2) contradicts S3-b*. We view such situations as in Figure 2 and also refer it as the *triangle conflict* in TAPS. This conflict level evolves or emerges mostly during the diagnostic stage which takes place in the teaching session. The beliefs and justifications involved in the belief revision process derived here can be used to remediate student's misconceptions for the particular problem he's translating.

Second, the student's beliefs (the misconception) can be deeply entrenched in his reasoning. The student may fail to be consistent over time due to the resurfacing of his tenacious misconceptions. This causes TAPS' beliefs that *"the student is capable of translating an algebra word problem"* to contradict with *"the student is incapable of translating an algebra word problem"* due to his resurfacing misconceptions. This is referred to as a **Level 2 conflict** (see Figure 3). TAPS' belief about the student's capability contradicts the student's performance over time. An observation compares the current set of problems that the student is doing with the previous problem sets he had gone through as indicated by the solid boxes in Figure 3. Such a situation occurs during the remediation stage of several problem sets. These entrenched misconceptions may be the same ones or different at each stage of observation. Level 2 is the meta level observation of the student's overall performance made by TAPS.

The notion of entrenchment

The term entrenched belief can be defined as a belief that is strongly bound in a web of observational and/or theoretical support (Brewer & Chinn, 1991). Such a belief is difficult to drop and be replaced by a new belief. This is even true when the new belief is inconsistent with the former one but results in a correct decision or answer, compared to the old belief which leads to an incorrect decision or answer. Entrenched beliefs are persistent. (Brewer & Chinn, 1991) suggested that to modify the entrenched belief one must encourage beliefs that have good observational support and that enter into a wide range

[2] In terms of conflict of beliefs, a student sometimes believes a concept C and $\neg C$ in a different context. This different context is as a hidden condition and usually leaves students unaware of the contradiction made.

of successful theoretical accounts of the world. Hence, in the case of students formulating the reversed equation due to their tenacious beliefs (misconceptions in this case), it is not an easy task to help the student overcome his deeply-seated misconceptions.

Dealing with entrenched beliefs

(Rosnick & Clement, 1981) have listed evidence that they used to identify student's lack of understanding when translating algebraic word problems. The behaviour that illustrated the entrenched misconceptions included: *i)* RC1 - the student reverts back to the reversed equation; *ii)* RC2 - the student accepts the correct equation but switches the meaning of the original problem; *iii)* RC3 - the student identifies the correct equation as being "weird" or "not making sense", and *iv)* RC4 - the student acknowledges that the correct equation "works" but states that he doesn't know why it works.

(Brewer & Chinn, 1991)'s study illustrates how people deal with information that violates their entrenched beliefs: *i)* BC1 - by rejecting the information; *ii)* BC2 - by reinterpreting the information; or *iii)* BC3 - by making partial changes in their former beliefs through accommodation and/or compartmentalisation.

TAPS combines both (Rosnick & Clement, 1981)'s justifications and (Brewer & Chinn, 1991)'s findings to diagnose a student's entrenched misconceptions. The integration is as follows: (RC1 & BC2), (RC2 & BC2), (RC3 & BC1) and (RC4 & BC3). Figure 4 is an example of (RC2 & BC2), where the student formulates the correct equation but he interprets the meaning of the original problem as its reverse, i.e. validating the reversed equation unconsciously.

Epistemic entrenchment in belief revision

The notion of entrenchment is also found in work on formal theories of Belief Revision such as (Gärdenfors, 1988) and (Berendt & Smaill, 1992). A belief revision system's function is to keep a knowledge base consistent in terms of the logical consequences of its contents (beliefs in this case). When there arises an inconsistency or a conflict between two beliefs, there ought to be a mechanism of resolving it. (Gärdenfors, 1988) introduced the notion of *epistemic entrenchment*[3] in resolving a conflict of beliefs. In a belief revision system with epistemic entrenchment, preference ordering over beliefs is employed when resolving a conflict. Each belief has an entrenchment value assigned. The higher the value, the more entrenched or valid it is to the system. The less tenacious belief of the two conflicting beliefs will be dropped, to maintain the consistency of beliefs in the system. TAPS employs such a belief revision system to resolve the two levels of conflicts that arise when the student commits errors related to the reversed equation.

Description of TAPS as a system

TAPS is intended to tutor students with problems in the *translation of algebraic problems*. The particular problem being dealt with is the student's deeply-seated or entrenched misconceptions. The aim is to assist the student to elicit his entrenched incorrect beliefs which are regarded as misconceptions and to replace such beliefs with correct beliefs.

Interacting with TAPS

There are three stages in the student's interaction with TAPS: pre-test, the teaching session and post-test. The pre-test comprises problems designed to elicit the misconceptions related to the reversal error. Statements are given. No feedback on the correctness of their answers are provided at this stage. It is designed to: *i)* determine the student's level of translation capability: below average, average, above average; *ii)* identify the student's strategy in translating the problem; *iii)* discover the student's misconceptions; and *iv)* assist in the evaluation of the student's progress i.e. as comparison to his post-test performance. In relation to the three mastery levels identified, our concentration is on the middle (average) mastery level where the student is partly correct in his translation process and partly incorrect. The student here

[3] "epistemic entrenchment is characterised by a complete preorder (a reflexive and transitive relation) over propositions which indicates which propositions are more valuable than others. This ordering influences revisions by the requirement that revisions retain more entrenched beliefs in preference to less entrenched ones. It may change over time or with the state of belief." (Doyle, 1992)

knows that it is a multiplication equation that he has to formulate but his misconceptions lead him to formulate a reversed multiplication equation.

After the pre-test, the teaching session starts. TAPS generates a dialogue session between TAPS and the student (multiple choice question-answer session). A belief revision system keeps track of both the student's and TAPS' consistencies in beliefs during the online session.

We applied two of the six teaching architectures described by (Schank, 1990) — the *Cascaded Problem Set* (CPS) and the *Responsive Question* (RQ) architectures. In relation to these teaching strategies, the questions and instructions used are carefully designed by considering possible student's reaction to entrenched beliefs and ways of provoking and entrapping student's beliefs. Questions which are isomorphic are asked repeatedly to ensure that their incorrect answers are not slips. In some questions, TAPS provide pertinent information such as knowledge about the mathematical notation (eg. *C is the number of children in the kindergarten*), and hints about visualising the problem (eg. *Given that there are 10 teachers, how many children are there?*) for the student to apply in his reasoning process while translating and formulating the algebra word problems. (Bernado & Okagaki, 1994) describe the information on the mathematical notation as *symbolic knowledge* and the hints as *problem-information context (PIC)*. (Bernado & Okagaki, 1994) also found that students are more likely to construct correct equations with the symbolic knowledge present, and those who received appropriate PIC did better in translating relational algebra word problems and constructing algebraic equations.

The post-test given after the teaching session is of a form isomorphic to the pre-test, and is used primarily for assessment of the student's performance. The results are compared to the pre-test performance in order to evaluate whether the teaching session has played a positive role in assisting the student.

The Belief Revision System in TAPS

A small but significant amount of research deals with belief revision in student modelling: *i)* (Huang, 1993) worked on the *logic of attention* to capture the student's inconsistent beliefs and attention. This logic could be applied in a Socratic tutoring system; *ii)* (Kono, Ikeda, & Mizoguchi, 1994) have developed *THEMIS*, a nonmonotonic induction student modelling which formulates contradictions that arise; *iii)* (Paiva, Self, & Hartley, 1994) applied a revision maintenance module in their *Theory and Application of General User Modelling System* to test techniques of learner modelling. Our work differs from theirs in that it deals with the student's entrenched beliefs that cause a contradiction in beliefs at the two levels of conflict, as discussed earlier. TAPS employs (Berendt, 1992)'s belief revision (BR) system with modifications. Berendt's BR system is a *Truth Maintenance System* (TMS) which combines the foundational and coherence approaches. The notion of epistemic entrenchment is applied to guide its decision in resolving a conflict. In BR, beliefs are modelled as statements in a restricted first-order logic. Its deductive process in this logic is modelled in a meta-theory as described in (Berendt & Smaill, 1992).

The BR system is used by TAPS to detect the contradictions that could occur (as in Figure 5) during the diagnosing and remediation session, and to provide the beliefs and justifications involved in the conflict. The justifications of the contradicting beliefs are used for explanation during the remediation session. (Kass & Finin, 1988) highlighted familiarity as a characteristic of understandable explanations. Hence, using student's own beliefs in the explanation provides information that he is familiar with. This could help his understanding in remediating his own misconceptions. More importantly, TAPS utilises BR's decision rule in resolving the contradiction, besides using its (TAPS) own decision rules.

Three categories of beliefs are involved in TAPS: *i)* the student's beliefs - obtained from his answers; *ii)* TAPS' own beliefs - generated from a sentence parser and some rules; and *iii)* the *Student Modelling* beliefs - the implication made by TAPS from *i)*. TAPS observation (for conflict Level 2) is also part of *SM*. Each of these categories has its entrenchment value: i) *Sev* - Student's entrenched value; ii) *Tev* - TAPS' entrenched value; iii) *Tov* - TAPS' observation's (meta level) entrenched value. The beliefs and their entrenchment values are added into the belief set as follows:

student_belief:- Sev is 0.9, add_assumption(answers(student,X) $\rightarrow$ X, Sev).

taps_belief:- Tev is 1, add_assumption(says(taps,X) $\rightarrow$ X, Tev).

taps_implication:- Tov is 1, add_assumption(X $\rightarrow$ Y, Tov).

```
T: Jean wants to prepare a home-made salad dressing.
   According to the recipe, she has to mix 3 times
   as much oil as vinegar.
   Does Jean has to put more oil or vinegar?

   a. vinegar      b. oil

L: b

T: Yes.
   Now, try formulate an algebraic equation of the
   statement "There is 3 times as much oil as vinegar
   in the salad dressing", using the letter L for the
   amount of oil and V for the amount of vinegar.

   a. 3L = V      b. 3V = L

L: b

T: Well done!
   You've formed the correct equation, this time.

   So, according to the equation 3V = L, if Jean puts
   a cup of oil, she will have to mix it with 3 cups
   of vinegar.
   Do you agree with that? Yes/No (y/n) ?

L: y
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Figure 4: A sample of the diagnosing session in TAPS

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>< >< ><  CONFLICT DETECTED  >< >< ><

Aha ... there's a conflict here. You seem to believe
   that Jean has to mix more vinegar than oil
as well as its opposite,
   that Jean has to mix more oil than vinegar
at the same time.
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Figure 5: A conflict detected by the BR system.

In the triangle conflict (level 1), the competition of beliefs is between *Tev* and *Sev*. *Tev* is always greater than *Sev* since ideally beliefs (knowledge) of TAPS about the domain are always assumed to be correct. Hence, the belief with *Tev* will be regarded as more valid and will survive in the competition, while the one with *Sev* will be dropped. BR will then notify TAPS that the disqualified belief and its justifications are the ones that the student has to withdraw from his reasoning process.

As mentioned before, level 2 conflict is where TAPS makes its meta level observations of the student's performance. However, the competition of the beliefs now is between two *Tov*s where both have the same entrenchment value. Berendt's BR system applies Gärdenfors' *conservative* way of choosing the belief that is to be kept when resolving a conflict: the earlier belief is considered to be more reliable compared with the new ones. Unfortunately, this is not the case during a teaching session. An earlier observation should not necessarily be assumed to be more valid than a recent one. In TAPS, we evaluate the abstract properties of the questions involved and refer these entities to the related *Tov*s. Among the entities evaluated for each question are: *i)* its level of difficulty; *ii)* whether it has been answered correctly or not by the student; and *iii)* the concept it carries (eg. the understanding of the coefficient's function in the equation; or mapping the keywords of the problem statement). The *Tov* that has weaker justifications out of this evaluation, will have its entrenchment value decremented and thus dropped from the belief set. The underlying mechanism for this evaluation has been further described in (Aziz, Pain, & Brna, 1995).

The following is a brief description of the underlying process of the example in Figures 4, 5 and 6. Figure 4 is a case of eliciting a student's entrenched belief. Here, the student translated to the correct equation. Through CPS and RQ teaching architectures, the third question is posed to determine whether the student understands the expression he had formulated. In other words, TAPS is testing if the student's misconceptions (which were detected previously) on *the concept of variable, equation and equal sign*

resurface. His answer here indicated that even though he had translated to the correct equation, he reverses the original meaning of the problem statement. The student's misconceptions on the *concept of variable, coefficient and equal sign* still persist in his mind.

An illustration of the underlying process is as follows.

B_S - Student's beliefs, with Sev as the entrenchment value

B_S1: Jean has to mix more oil than vinegar

B_S2: the equation is 3V = L

B_S3: 3V = L means that for a cup of oil, mix 3 cups of vinegar

B_T - TAPS' own beliefs, with Tev as the entrenchment value

B_T1: mix more oil than vinegar

B_T2: the equation is 3V = L

B_T3: 3V = L means that for a cup of oil, mix 1/3 cup of vinegar

SM - The Student Modelling, i.e TAPS' implications of the student's beliefs, with Tov as the entrenchment value

SM1: $B_S1 \rightarrow$ mix more oil than vinegar

SM2: $B_S2 \rightarrow$ the amount of oil is more than vinegar

SM3: $SM2 \rightarrow$ mix more oil than vinegar

SM4: $B_S3 \rightarrow$ mix more vinegar than oil

SM5: $SM4 \rightarrow$ negate(mix more oil than vinegar)

Hence, a contradiction between *"mix more oil than vinegar"* and *"negate(mix more oil than vinegar)"* occurs as shown in Figure 5. BR (through its decision rule) resolves the conflict by remaining *"mix more oil than vinegar"* and its justifications in the belief set, and dropping *"negate(mix more oil than vinegar)"* and its justifications from the belief set. TAPS will be notified by BR, and all the justifications involved will be *handed* over to TAPS to be used as part of the explanation to the student. See Figure 6.

Discussion and Conclusion

The current system, TAPS is a prototype to demonstrate the idea of weaving a belief revision system with the epistemic entrenchment notion in it together with a domain where many students have entrenched misconceptions. It is implemented in SICStus Prolog and currently manages *i)* to generate TAPS' own beliefs based on system-student interactions for ten sets of word problems; *ii)* to form the student's beliefs from his answers; *iii)* to detect and resolve both levels of conflict; *iv)* to use the justifications which support the contradicting beliefs to provide remediation to the student. From a preliminary formative internal evaluation by the first author using the ten sets of algebra word problems, the initial prototype of TAPS shows some positive contributions of a belief revision system to a tutoring system such as: detecting conflicts to trigger the system; providing the contents to construct an explanation, and assisting the system in its meta level observation of the student's performance. A formative external evaluation of the system in use is planned to take place among students and teachers to obtain feedback on the educational benefits of using the system.

The ideas illustrated in this paper are not only beneficial for the reversal error in algebraic word problems but also contribute to student modelling in several ways: *i)* they cater for student's unstable performance i.e the system has to handle fluctuating student's understanding or mastery levels; *ii)* they can be applied in other domains where student's misconceptions are resilient and tend to resurface such as in the domains of relativity and quantum mechanics (Brewer & Chinn, 1991) and several mathematics and science concepts mentioned by (Murray, Schultz, Brown, & Clement, 1990); *iii)* they are relevant also for students with poor performance in subject materials which he finds difficult to master.

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T: Here is the explanation of how the conflict occurs.

   The belief 'mix more oil than vinegar', arises from:

   1. Your answer - 'Jean has to mix more oil than vinegar'
      leads to the belief  'mix more oil than vinegar'.
   2. TAPS' belief - 'mix more oil than vinegar'

   The negation of the belief 'mix more oil than vinegar',
   arises from:

   1. Your answer - 'for 1 cup of oil mix 3 cups of vinegar'
      leads to the belief that 'mix more vinegar than oil'
      which then leads to the belief
      'negate(mix more oil than vinegar)'

   The belief 'mix more oil than vinegar' and its justifications
   are correct
   but its opposite belief i.e 'negate(mix more oil than vinegar)'
   and the initial cause of it which is
   your answer - 'for 1 cup of oil mix 3 cups of vinegar'
   has to be fixed.

   Now, do you want to
   a. get more explanation of why this answer is incorrect
   b. fix your answer

L: a

T: The letters L and V in equation formulated, 3V = L
   from the problem statement do not stand for the physical
   object.
   That is L doesn't represent the oil as the exact ingredient
   for the salad dressing, and V does not stand for the vinegar.
   L is a variable that represents a number,
   Hence in this problem, L is the amount of oil ...

   < Explanation on the concepts of variable, coefficient and
    equal sign.
    This is retrieved from a partial canned text explanation >
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Figure 6: A sample of the remediation session in TAPS