

Lecture 14: Edge Detection (cont'd)

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Reading

SH&B, 4.3.4–4.3.5

14.1 Second-Derivative Zero Crossings and Gradient Magnitude

One caution should be added to our discussion of zero-crossings of second derivatives (e.g., the Laplacian) as an edge detector. Remember that the idea is to use zeroes of second derivatives to indicate maxima of gradient magnitude. The problem is that this doesn't include any notion of *how strong* that maximum is. In other words, even a small blip in the gradient magnitude is labelled an edge. We can remedy this by combining second- and first-order derivatives. In other words, an edge can be defined as places where

$$\nabla^2 f = 0 \text{ and } \nabla f > T$$

Of course, now we lose one of the greatest virtues of zero crossings: their continuity. One way to deal with such breaks in continuity is to slowly drop the threshold in places where there is clearly a break. Your text refers to this as *hysteresis thresholding*. In other contexts, it is sometimes known as *edge relaxation*.

14.2 Scale Space

Section 4.3.4 briefly introduces the notion of a *scale space*. This is the space of measurements that you get after blurring an image by successively larger and larger Gaussian kernels. The essential idea of scale spaces is this:

1. It is impossible to make an infinitely small measurement—all measurements of the physical world involve an *aperture*.
2. The size of the aperture affects the measurement.
3. In a sense “information” exists at different aperture scales.
4. To extract information at different scales, you must measure at those scales (or simulate such).
5. *Smaller (“more precise”) apertures don't necessarily produce more information than larger ones.*

In a sense, what we've been doing all along by blurring images to smooth away noise is to make measurements at large scales (with larger apertures). By doing so, we get above the scale of noise and try to make measurements at scales more natural for the objects in the image. Of course, if the objects are small relative to the noise (or the noise is large relative to the objects of interest), we can't separate them.

Moral: Choose the right scale for the information you’re trying to extract.

We will come back and talk about this idea more in Lecture 22. For now, it is sufficient to think only about some edges being more accurately detected at scales larger than single-pixel changes. This idea, plus another important one from differential geometry, is illustrated in the following edge detection algorithm.

14.3 Canny Edge Detector

14.3.1 Basic Definition

One problem with Laplacian zero-crossings as an edge detector is that it is simply adding the principal curvatures together—it doesn’t really determine a maximum of gradient magnitude.

The *Canny Edge Detector* defines edges as *zero-crossings of second derivatives in the direction of greatest first derivative*.

Let’s examine each of the parts of this definition:

The Direction of Greatest First Derivative This is simply the gradient direction (w in first-order gauge coordinates).

The Second Derivative in The Direction of ... We can compute this using the matrix of second derivatives (Hessian):

$$L_u = u^T H u$$

Zero Crossings of... As with the Marr-Hildreth edge detector, we’ll use positive-negative transitions to “trap” zeroes.

Thus, in gauge coordinates, the Canny detector is

$$L_{ww} = 0$$

In terms of x and y derivatives, this can be expanded to

$$L_{ww} = \frac{1}{L_x^2 + L_y^2} \begin{bmatrix} L_x & L_y \end{bmatrix} \begin{bmatrix} L_{xx} & L_{xy} \\ L_{yx} & L_{yy} \end{bmatrix} \begin{bmatrix} L_x \\ L_y \end{bmatrix} = 0 \quad (14.1)$$

Zero-crossings in this measure give connected edges much like the Laplacian operator but more accurately localize the edge.

14.3.2 Scale and the Canny Operator

The Canny algorithm also makes use of multiple scales to best detect the edges. In particular, it uses Gaussian blurring kernels of varying sizes $\{\sigma_k\}$ and selects the one that is most stable. The selection of the best scale at which to make the measurement is the subject of much research, including current research.

Your text writes this as

$$\frac{\partial}{\partial \bar{n}} (G * g) = 0$$

where g denotes the image, \bar{n} denotes the edge normal (the gradient direction), and G denotes a Gaussian blurring kernel. Remember, though, this can be rewritten as

$$\frac{\partial G}{\partial \bar{n}} * g = 0$$

This is a nice, compact notation, but remember that \bar{n} is also determined from the blurred image $G * g$. This notation also leaves out the details in Eq. 14.1.

If we use Eq. 14.1 to determine the edges, we can use Gaussian blurring by measuring each of the five derivatives in Eq. 14.1 using derivatives-of-Gaussians of the appropriate standard deviation.

14.3.3 The Algorithm

Your text describes the algorithm in this way [my comments added]:

1. Repeat steps (2) till (6) for ascending values of the standard deviation σ .
2. Convolve an image g with a Gaussian of scale σ .
[Remember that this step is not explicitly necessary—simply convolve with derivatives of Gaussians when performing differentiation.]
3. Estimate local edge normal directions \bar{n} using equation (4.61) for each pixel in the image.
[Remember: \bar{n} is the gradient direction.]
4. Find the location of the edges using equation (4.63) (non-maximal suppression).
[i.e., find the zero crossings]
5. Compute the magnitude of the edge using equation (4.64).
[i.e., compute the gradient magnitude as well]
6. Threshold edges in the image with hysteresis to eliminate spurious responses.
[Threshold the gradient magnitude as discussed in Section 14.1 earlier in this lecture.]
7. Aggregate the final information about edges at multiple scale using the ‘feature synthesis’ approach.
[Select the “best” scale.]

Vocabulary

- Scale space
- Canny edge detector